Quantifying and Improving Transferability in Domain Generalization

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Main result

We propose an evaluation metric for measuring the transferability in domain generalization. Based on this metric, we design algorithms to further improve out-of-domain generalization over SoTA methods.

Transfer learning

Source domain \( \delta \)

Embedding \( g \)

Classifier \( h \)

Guitar!

Target domain \( \mathcal{T} \)

Embedding \( g' \)

Classifier \( h' \)

Piano!

How do we measure the transferability of feature representations?

Transferable:

Every near-optimal source classifier is also near-optimal on the target

\[
\arg\min_{\mathcal{H}} (c_2) := \{ h \in \mathcal{H}, c_2(h) \leq \inf_{h' \in \mathcal{H}} c_2(h') + \delta \}
\]

Definition: \( \delta \) is \((\delta_1, \delta_2)\)-transferable to \( \mathcal{T} \) if

\[
\arg\min_{\mathcal{E} \times \mathcal{D}} (\epsilon_2, \delta) \subseteq \arg\min_{\mathcal{E} \times \mathcal{D}} (\epsilon_2, \delta)
\]

Use Transfer Measures to quantify transferability

Source domain \( \delta \)

Classifier \( h \)

Excess risk

\[
\epsilon_{2}(h) := c_2(h) - \inf_{h' \in \mathcal{H}} c_2(h')
\]

Target domain \( \mathcal{T} \)

Classifier \( h' \)

Excess risk

\[
\epsilon_{2}(h') := c_2(h') - \inf_{h' \in \mathcal{H}} c_2(h')
\]

Transferable \( \Rightarrow \) Small transfer measure, if \( \Gamma = \arg\min(c_{2}, \delta) \).

Theoretical results

Generalization bound (tighter than the one using \( \mathcal{H} \)-divergence)

\[
\epsilon_{2}(h) \leq \epsilon_{2}(h') + \tau_{2}(\delta, \mathcal{T}) + \epsilon_{2}(h) - \epsilon_{2}(h') \quad \text{if} \quad \epsilon_{2}(h) \leq \epsilon_{2}(h') + \tau_{2}(\delta, \mathcal{T})
\]

If the optimal errors are zero, transfer measure with 0-1 loss is equivalent to total variation when \( \Gamma = \mathcal{H} \), includes all binary classifiers

Total variation

\[
d_{TV}(\delta, \mathcal{T}) = \sum_{i \in \mathcal{D}} \left| p_{\delta}(x, y) - p_{\mathcal{T}}(x, y) \right| dx
\]

Two domains can still be transferable even if the joint distributions are dissimilar

Invariance principles (error depends on both the marginal and conditional distributions)

- Transferability: invariance of excess risks (joint)
- H-divergence: invariance of feature distributions \( p(\cdot) \) (marginal)
- Invariant risk minimization (IRM): invariance of the optimal predictors (conditional)
- Generalized label shift: invariance of feature distributions within the same class \( p(z|\cdot) \) (conditional)

How to Compute Transfer Measures?

We make the following approximations

- We use surrogate loss to approximate 0-1 loss
- We use the final loss after training to approximate the optimal loss
- We use a neighborhood in the parameter space to approximate the minimal set
- The resulting transfer measure is a lower bound, but we can still use it to refute transferability

Final result as a lower bound of transfer measure

\[
\sup_{\theta \in \mathcal{D} \in \mathcal{L}} \epsilon_{2}(h) - \epsilon_{2}(h') \quad \text{if} \quad \epsilon_{2}(h) \leq \epsilon_{2}(h') + \tau_{2}(\delta, \mathcal{T})
\]

\( \theta \) parameter of classifier

\[
\hat{h} = \arg\min_{\theta} \left\{ \epsilon_{2}(h) - \epsilon_{2}(h') \right\}
\]

The learned classifier after training

If the lower bound is large, then it is not transferable

Algorithms

Domain Generalization (DG) learns feature embeddings and classifiers from several source domains

Source domains

- Art
- Cartoon
- Sketch
- Photo

Target domain

- Guitar
- Piano

Suppose we have \( n \) source domains: \( \delta_1, \ldots, \delta_n \) and a target domain \( \mathcal{T} = \delta_0 \)

Evaluate transferability

\[
\max_{i \in [n]} \max_{s \in \delta_i} \max_{t \in \delta_0} \epsilon_{2}(h + g) - \epsilon_{2}(h + s)
\]

Improve transferability

\[
\min_{i \in [n]} \frac{1}{n} \sum_{s \in \delta_i} \epsilon_{2}(h + g) + \max_{s \in \delta_i} \max_{t \in \delta_0} \epsilon_{2}(h + g) - \epsilon_{2}(h + t)
\]

Experiments

- Many popular algorithms are not learning transferable features: a near-optimal source classifier is not near-optimal on the target
- Our Transfer algorithm learns more transferable features than ERM, DANN, IRM, GroupDRO, etc.
- Best performers: Transfer (ours), CORrelation Alignment (CORAL), Spectral Decomposition (SD)

References

- Ben-David et al, “A theory of learning from different domains,” Machine Learning, 2010

Full talk: https://www.youtube.com/watch?v=Ce3PyHA54GI