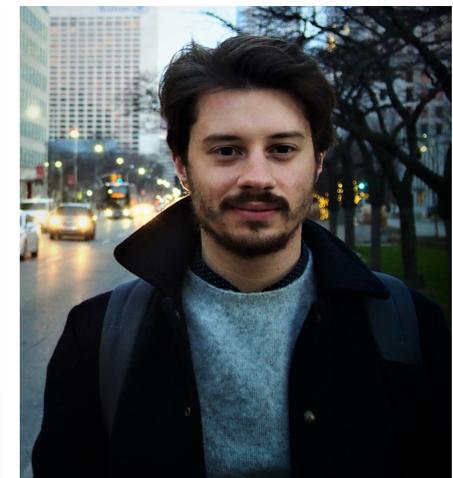
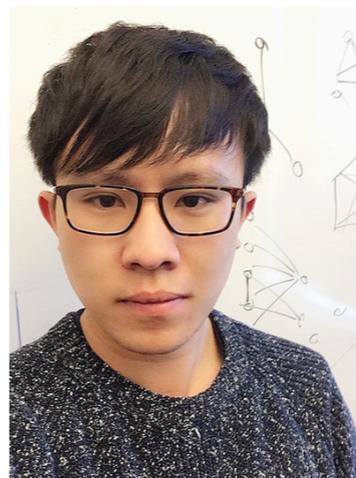


On Learning Language-Invariant Representations for Universal Machine Translation

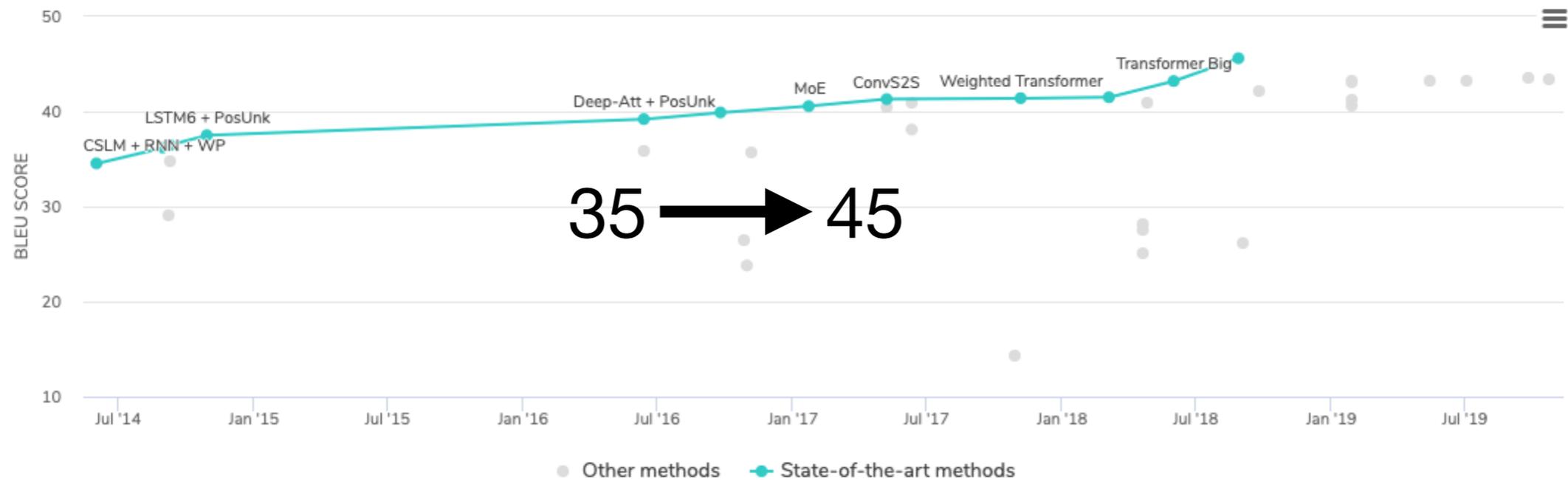
Han Zhao, Junjie Hu, Andrej Risteski
{han.zhao, junjieh, aristesk}@cs.cmu.edu

Carnegie Mellon University



Recent Success of Neural Machine Translation

Machine Translation on WMT2014 English-French



Machine Translation, ~3M parallel sentences [Cho et al. 2014; Devlin et al. 2014]

Neural Machine Translation is Data Hungry

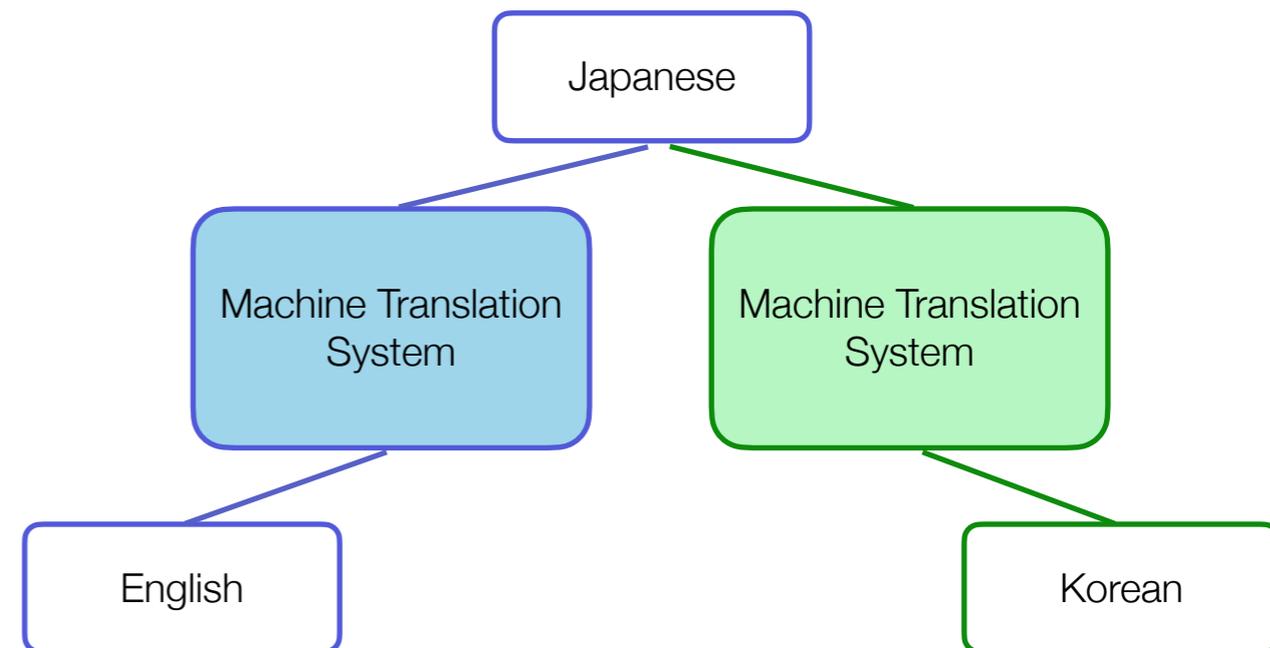
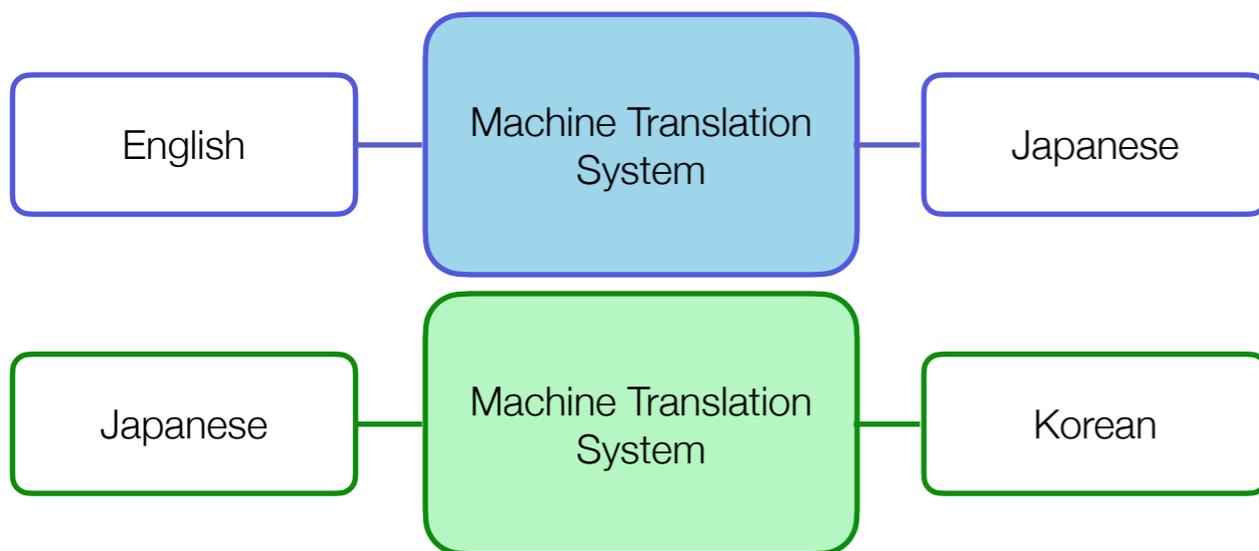


Figure from [Gu et al. 18]

Source	Target	Corpora size	BLEU Scores
English	French	~3M	~40
English	German	~1.92M	~35
Finnish	English	~1.96M	~34
Romanian	English	~400K	~30

Typical Pipeline of Multilingual Machine Translation

- **Separate MT systems:** Hard to maintain all systems
- **Pivot methods:** src-to-pivot & pivot-to-tgt translations



Machine translation by triangulation: Making effective use of multi-parallel corpora, [Cohn et al 07]

Cross-Lingual Representations by Neural Models

- **Language similarity:** similar words, grammar, order.
- **Shared space:** learning word/sentence representations jointly

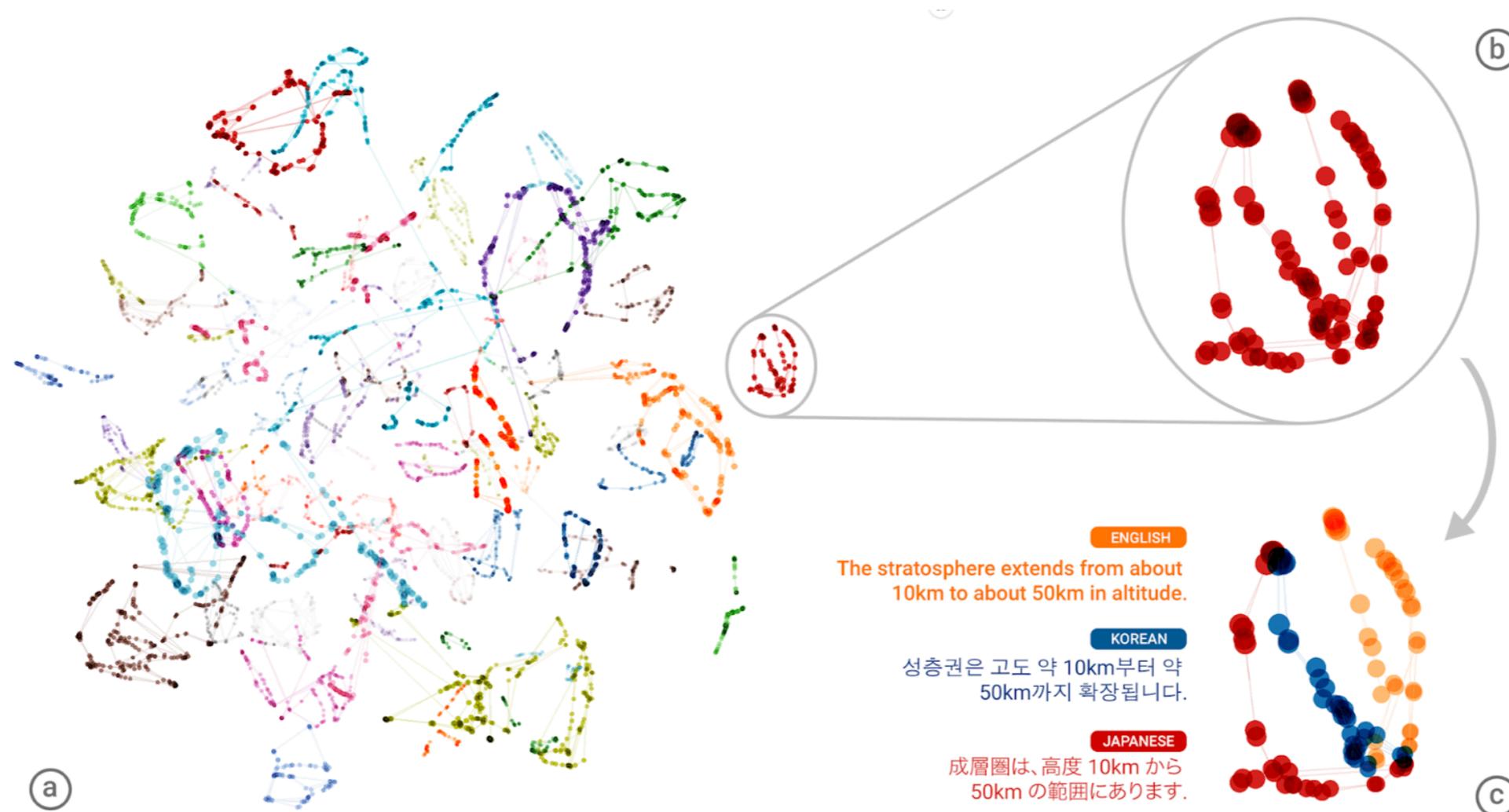
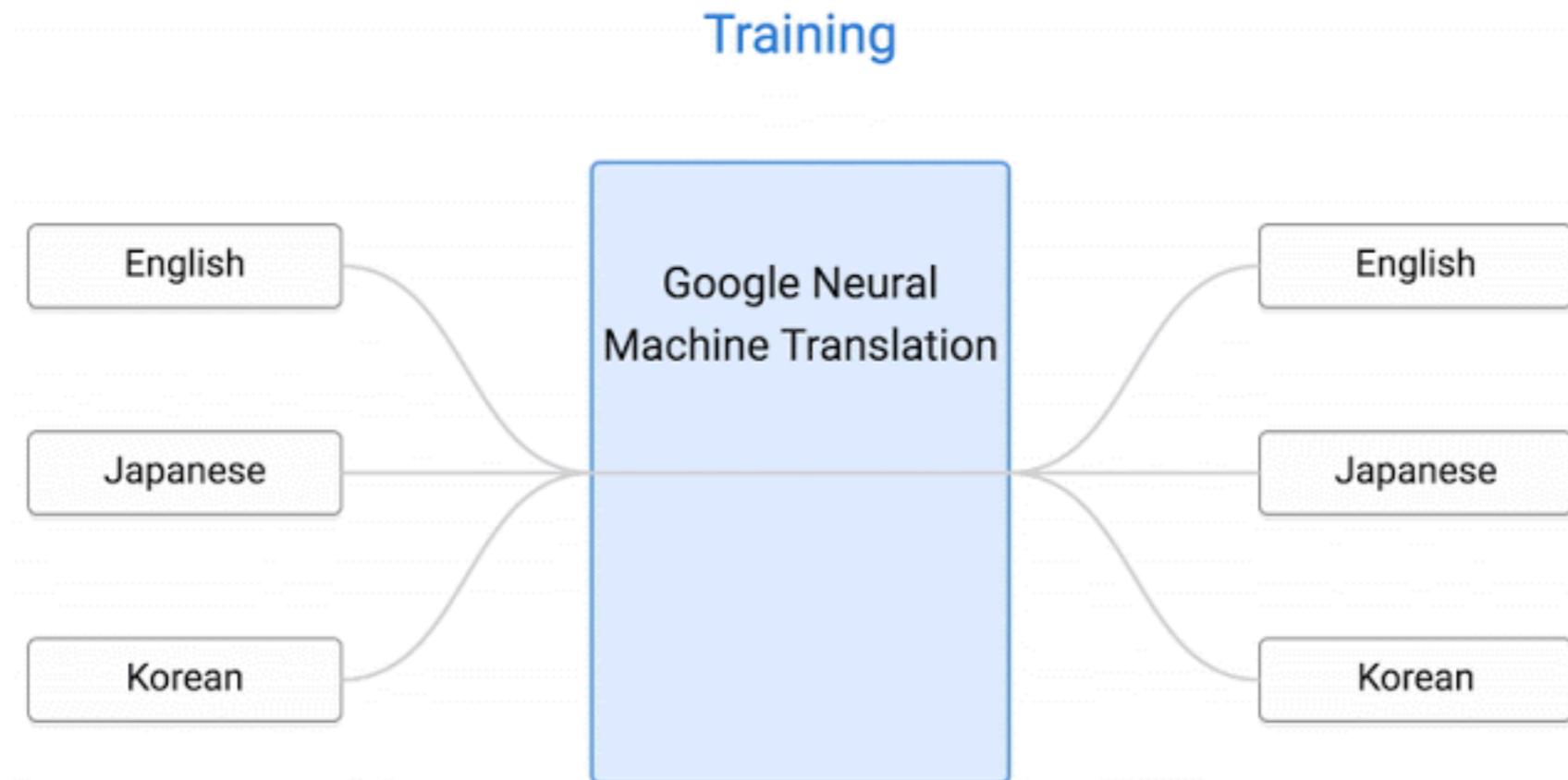


Photo credit: <https://ai.googleblog.com/2016/11/zero-shot-translation-with-googles.html>

Why Universal Machine Translation (UMT)?

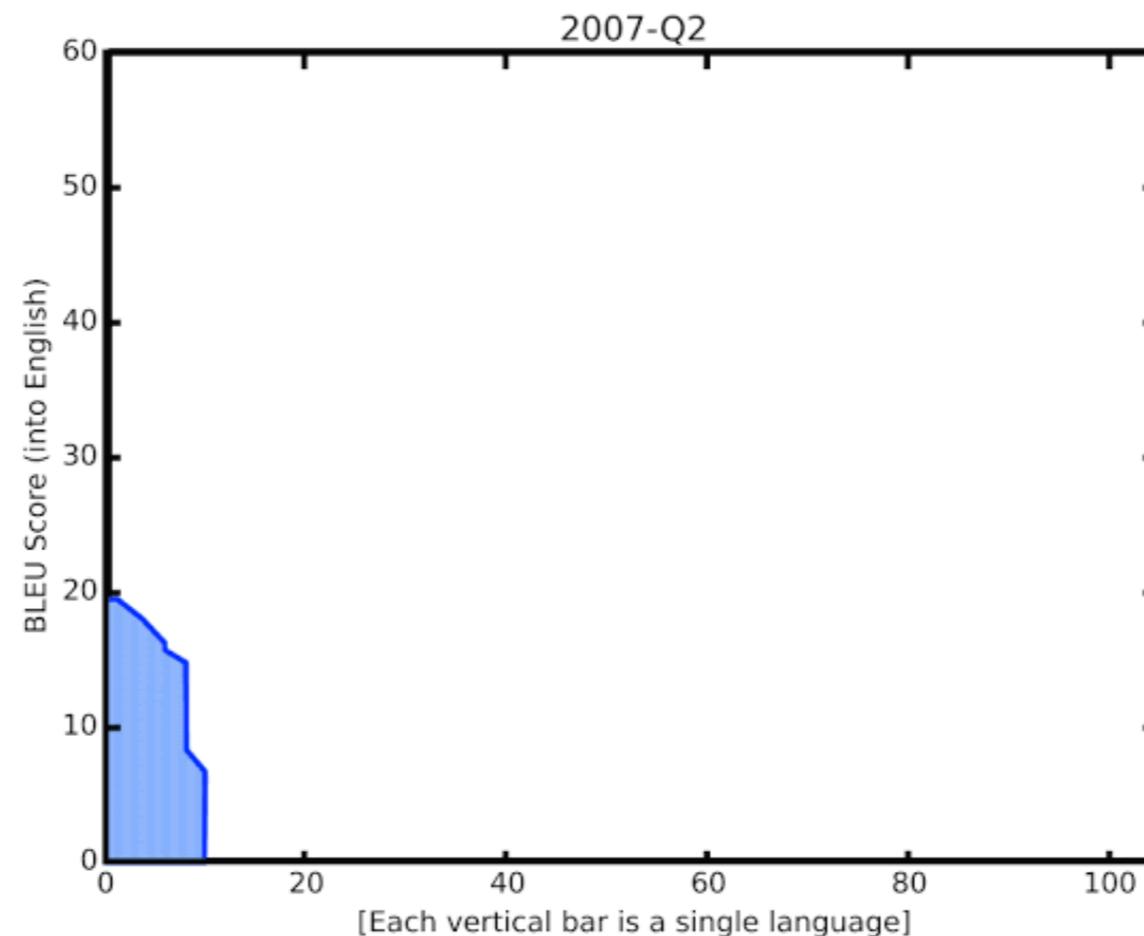
- **Single model:** many-to-one, one-to-many
- **Zero-shot translation:** improve low-resource translation



Johnson et al. Google's Multilingual Neural Machine Translation System: Enabling Zero-Shot Translation, TACL 2017.

Recent Advances of UMT

- **Language coverage:** 100+ languages in Google's M4
- **Web-mined data:** 25 billion examples
- **Quality:** +5 BLEU score over all 100+ languages

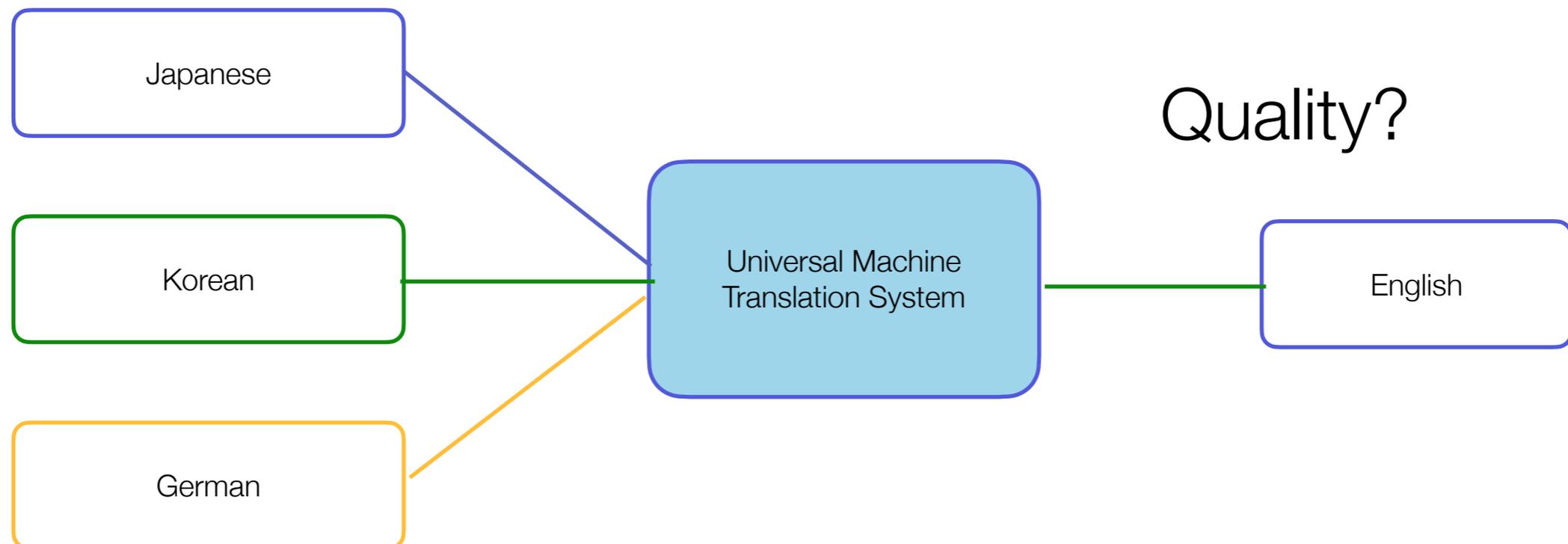


Massively Multilingual Neural Machine Translation in the Wild: Findings and Challenges [Arivazhagan et al. 19]

Challenge: Theoretical Understanding of UMT

Despite the empirical success, theoretical understanding is only nascent

- **Translation Error:** Is there a performance limit even with unlimited amount of computation & data
- **Sample Complexity:** How many language pairs are required to train UMT?



Challenge: Theoretical Understanding of UMT

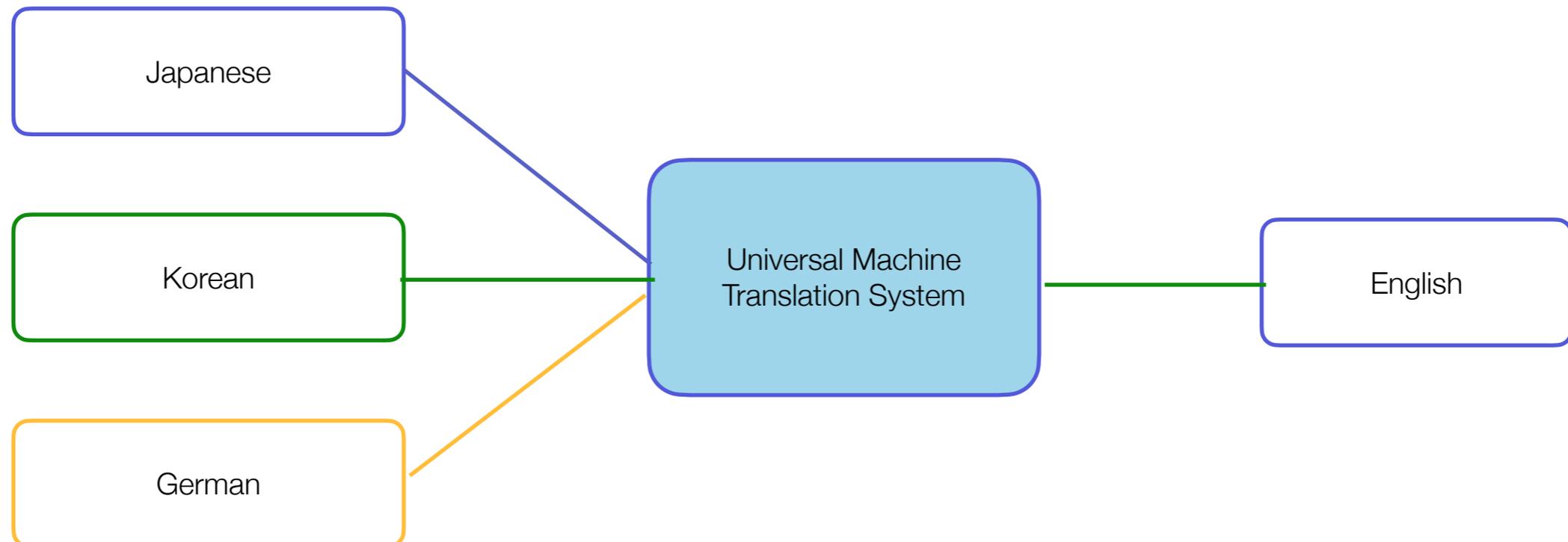
Despite the empirical success, theoretical understanding is only nascent

- Translation Error: Is there a performance limit even with unlimited amount of computation & data
 - Without assumption on the parallel corpus used for training, **at least one** translation task has to incur a large error
- Sample Complexity: How many language pairs are required to train UMT?
 - Under an encoder-decoder generative assumption of the data, a **linear** number of translation pairs suffice for the purpose of UMT

A Theoretical Model for UMT

Let $\mathcal{L} = \{\text{English, French, German, Chinese, ...}\}$ be the set of all languages of interest.

- For each $L \in \mathcal{L}$, we associate with L an alphabet Σ_L
- A sentence x in L is a sequence of symbols from Σ_L , i.e., $x \in \Sigma_L^*$
- For a pair of languages L, L' , we use $\mathcal{D}_{L,L'}$ to denote the joint distribution over the parallel sentence pairs from L and L'



A Theoretical Model for UMT

Problem Setting:

- For each pair of languages L, L' , there exists a **true translator**

$$f_{L \rightarrow L'}^* : \Sigma_L^* \rightarrow \Sigma_{L'}^*$$

- Given a translator f from L to L' , we use the 0-1 loss to measure the translation quality w.r.t. the true translator:

$$\text{Err}_{\mathcal{D}}^{L \rightarrow L'}(f) := \mathbb{E}_{\mathcal{D}}[\ell(f(X), f_{L \rightarrow L'}^*(X))]$$

where $\ell(x, x') = 0$ iff $x = x'$.

There exists a perfect translator that translates input sentence from any language to a target language L :

$$f_L^*(x) = \sum_{L' \in \mathcal{L}} \mathbb{I}(x \in \Sigma_{L'}^*) \cdot f_{L' \rightarrow L}^*(x)$$

Can we recover the perfect translator through UMT?

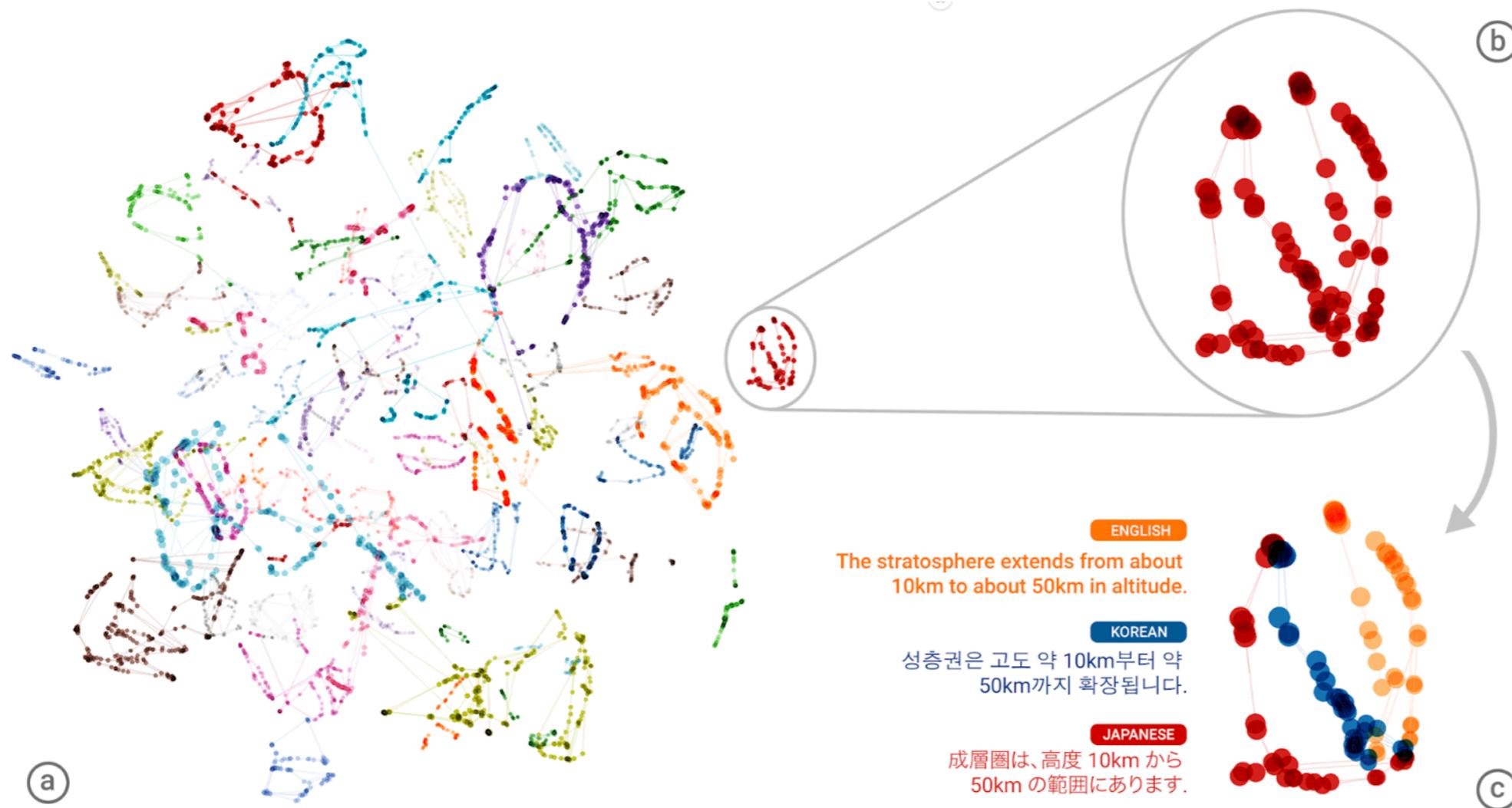
Universal Machine Translation

Universal Language Mapping:

A function mapping $g : \bigcup_{i \in [K]} \Sigma_{L_i}^* \rightarrow \mathcal{Z}$ is called **universal** if

$$g_{\#} \mathcal{D}_i = g_{\#} \mathcal{D}_j, \forall i \neq j$$

Different languages have the same distribution under representation \mathcal{Z}



An Impossibility Theorem

A simple warm-up (Two-to-One):

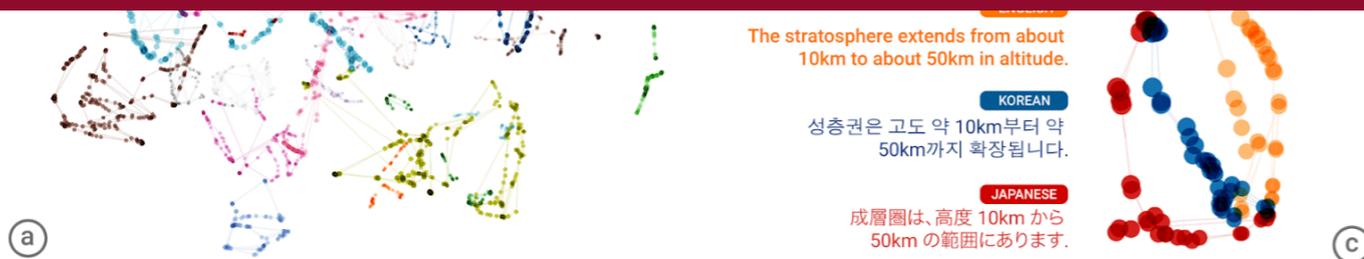
Theorem (informal): Consider a restricted setting of universal machine translation task with two source languages and one target language. If g is a universal language mapping, then for any decoder $h : \mathcal{Z} \rightarrow \Sigma_L^*$,

$$\text{Err}_{\mathcal{D}_0}^{L_0 \rightarrow L}(h \circ g) + \text{Err}_{\mathcal{D}_1}^{L_1 \rightarrow L}(h \circ g) \geq d_{\text{TV}}(\mathcal{D}_{L_0, L}(L), \mathcal{D}_{L_1, L}(L)).$$

Translation errors from L_0, L_1 to L

Distance between sentence distributions over L

Uncertainty Principle: UMT has to make a large error on at least one translation task



An Impossibility Theorem

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Translation errors from L_0, L_1 to L

Distance between sentence distributions over L

- This is an information-theoretic lower bound, i.e., algorithm-independent
- The theorem still holds even if we use different encoders for different languages, but wouldn't hold any more if we use target-dependent decoder!
- The lower bound gets larger whenever target data are dissimilar between different translation tasks

An Impossibility Theorem

In general (Many-to-One):

Maximum Translation Error:

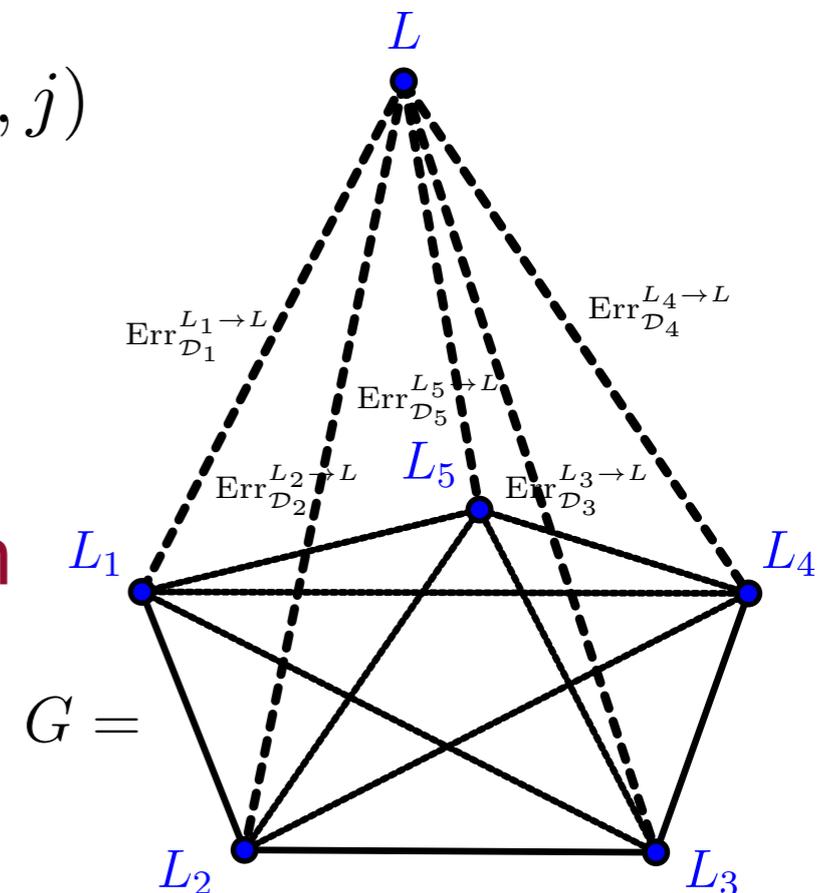
$$\max_{i \in [K]} \text{Err}_{\mathcal{D}_i}^{L_i \rightarrow L}(h \circ g) \geq \frac{1}{2} \max_{i \neq j} E(i, j)$$

Average Translation Error:

$$\frac{1}{K} \sum_{i \in [K]} \text{Err}_{\mathcal{D}_i}^{L_i \rightarrow L}(h \circ g) \geq \frac{1}{K(K-1)} \sum_{i < j} E(i, j)$$

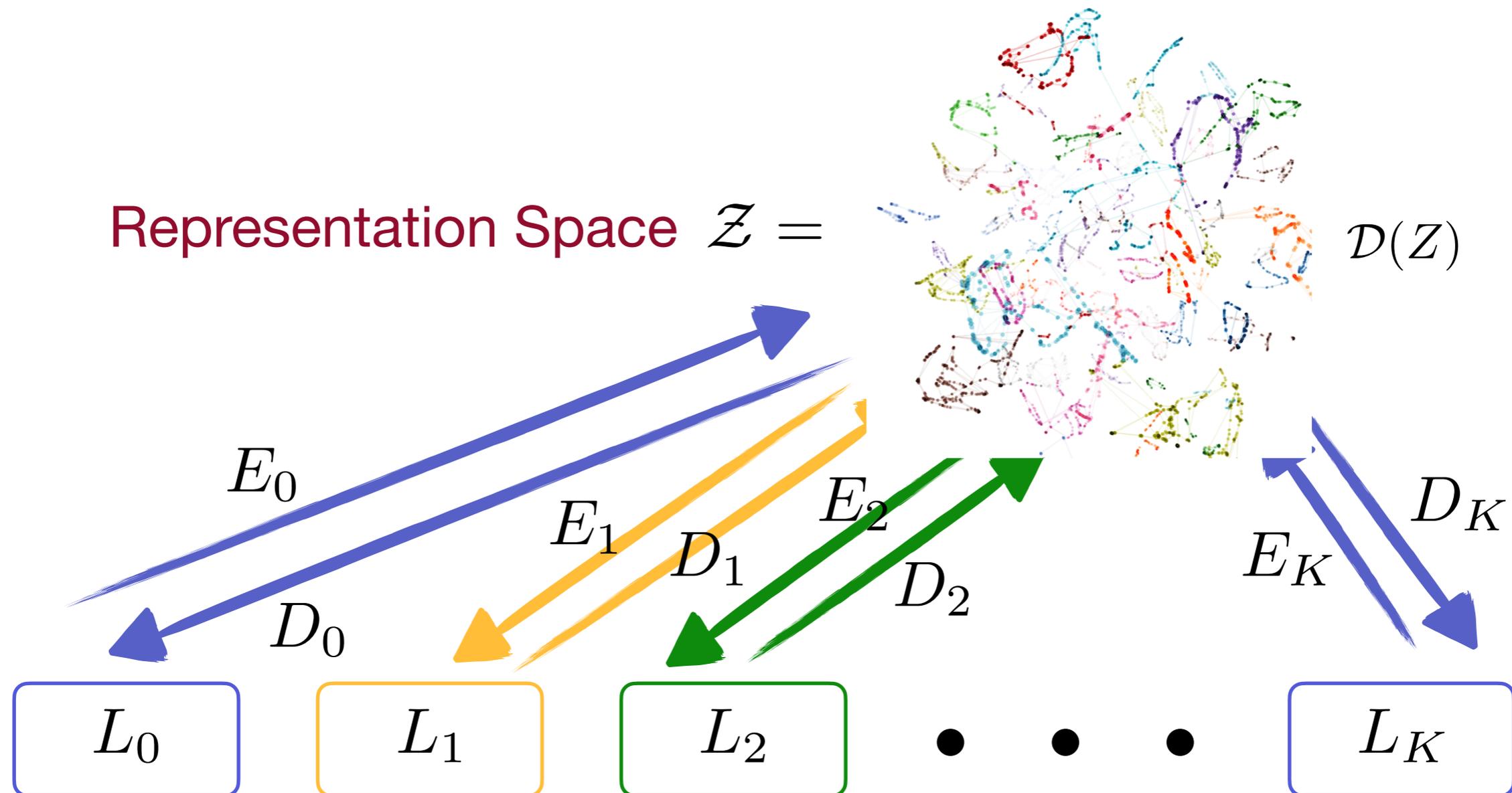
$E(i, j)$ measures how different two translation tasks are:

$$E(i, j) := d_{\text{TV}}(\mathcal{D}_{L_i, L}(L), \mathcal{D}_{L_j, L}(L))$$



A Generative Model of UMT

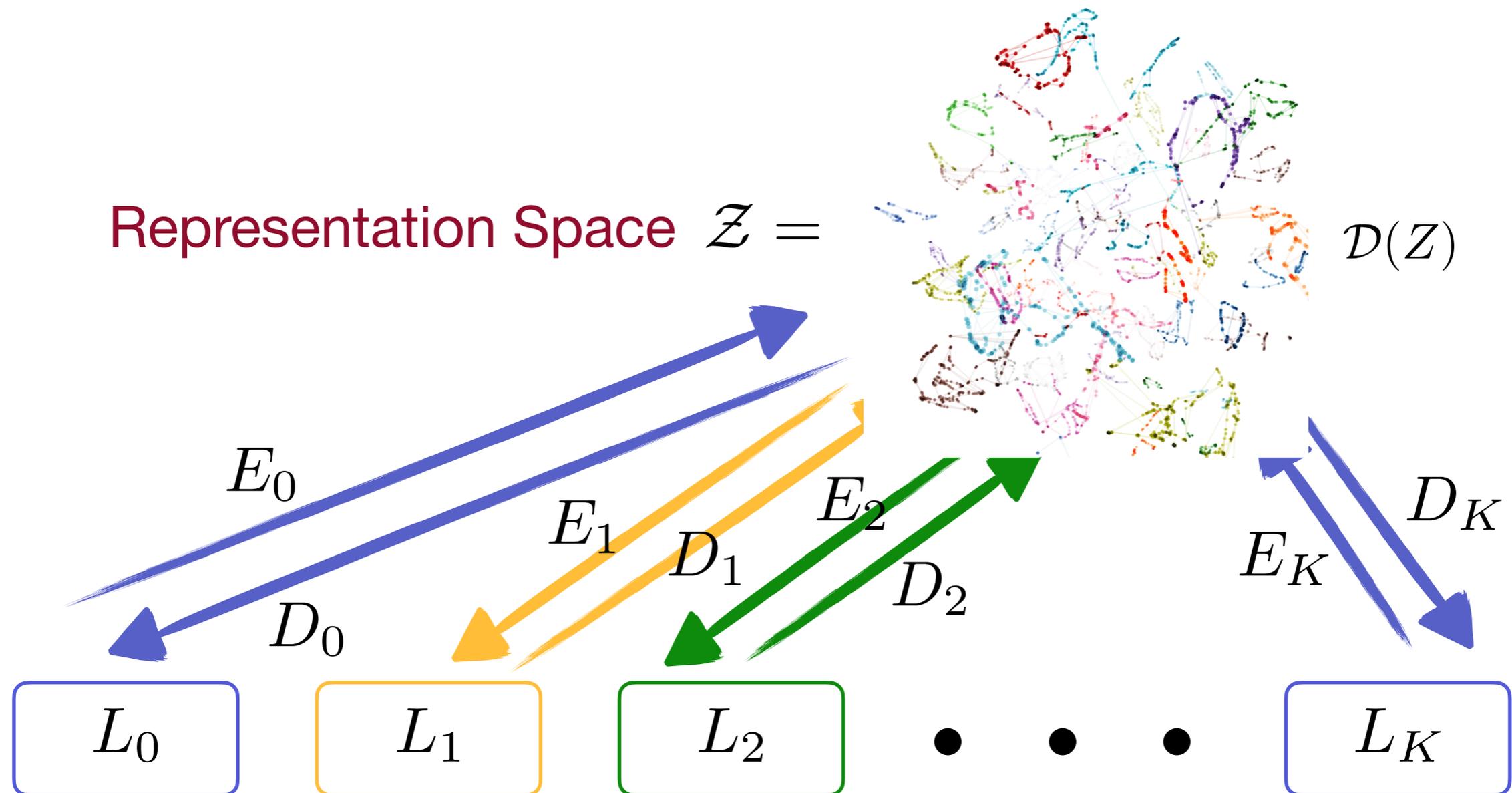
The impossibility theorem holds in the worst case without any assumption on the data generating distribution of parallel corpus. What if we assume an encoder-decoder generative process?



We assume that $E_k \in GL_d(\mathbb{R})$, $D_k = E_k^{-1}$, $\forall k \in [K]$

A Generative Model of UMT

Why this assumption on data generative process helps?



$$\forall i, j \in [K], \quad \text{Err}_{\mathcal{D}_i}^{L_i \rightarrow L}(h \circ g) + \text{Err}_{\mathcal{D}_j}^{L_j \rightarrow L}(h \circ g) \geq d_{\text{TV}}(\mathcal{D}_{L_i, L}(L), \mathcal{D}_{L_j, L}(L)) = 0$$

The lower bound still holds, but it gracefully reduces to 0 under this encoder-decoder assumption on data generative process.

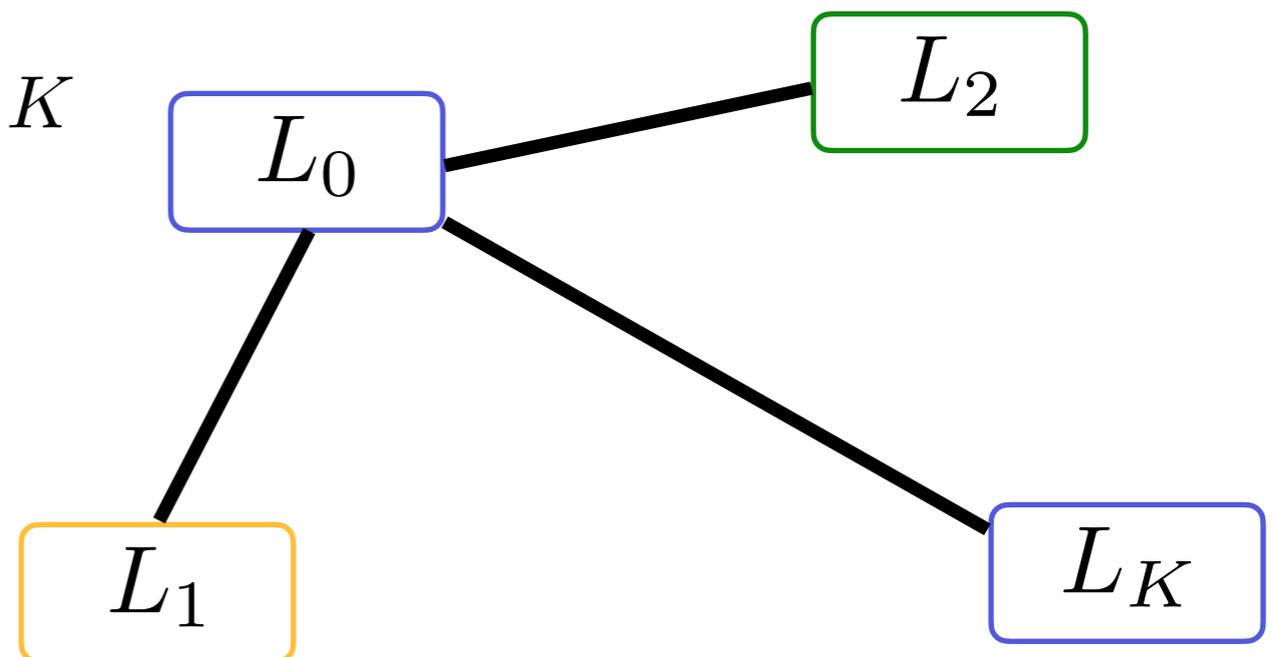
How Many Language Pairs Suffices?

Naively, one might think we need $\Omega(K^2)$ language pairs, one for each pair.

Our result: under some mild assumptions, only a linear number $O(K)$ of translation pairs suffices!

Translation Graph: H

- Each node = a language
- Two nodes are connected if we see the corresponding translation pair
- H is assumed to be **connected**: we need to see every language at least once
- The diameter of H is bounded by K



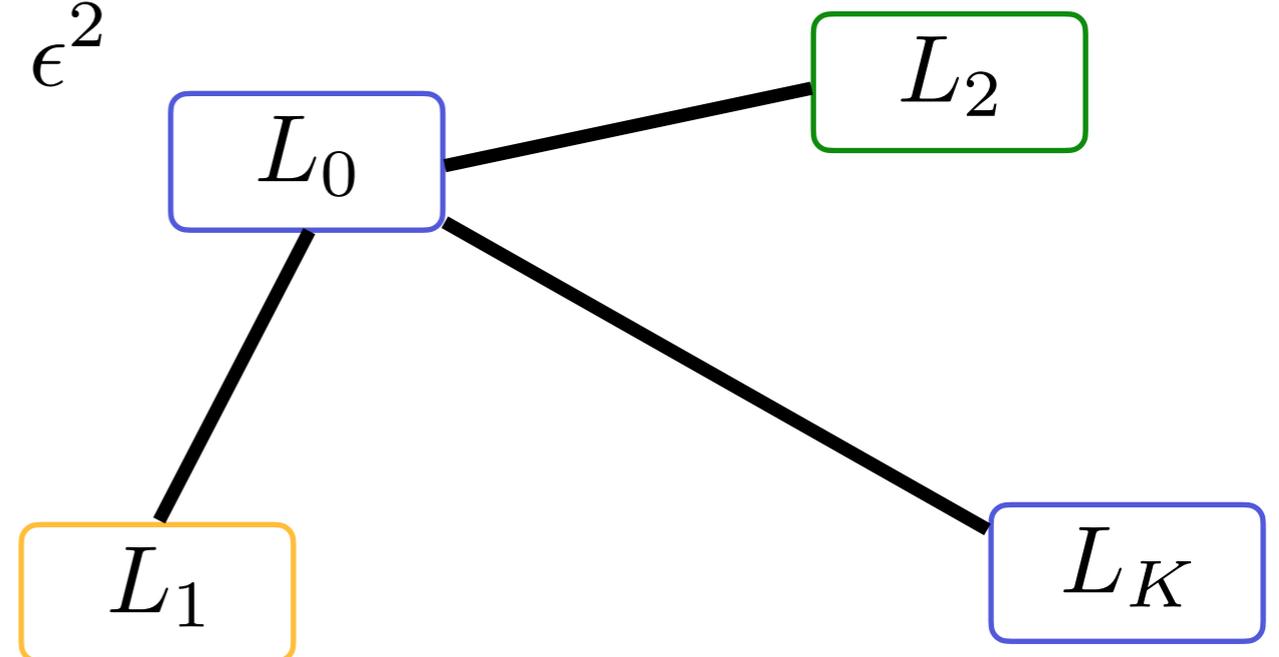
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Translation Graph: H

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- The diameter of H is bounded by K

Theorem (informal): Let $\text{diam}(H)$ be the diameter of the translation graph H , then for any pair of language L_i, L_j , the translation error has the following upper bound:

$$\varepsilon(\hat{E}_i, \hat{E}_j) \leq \rho \cdot \text{diam}(H) \cdot \epsilon^2$$

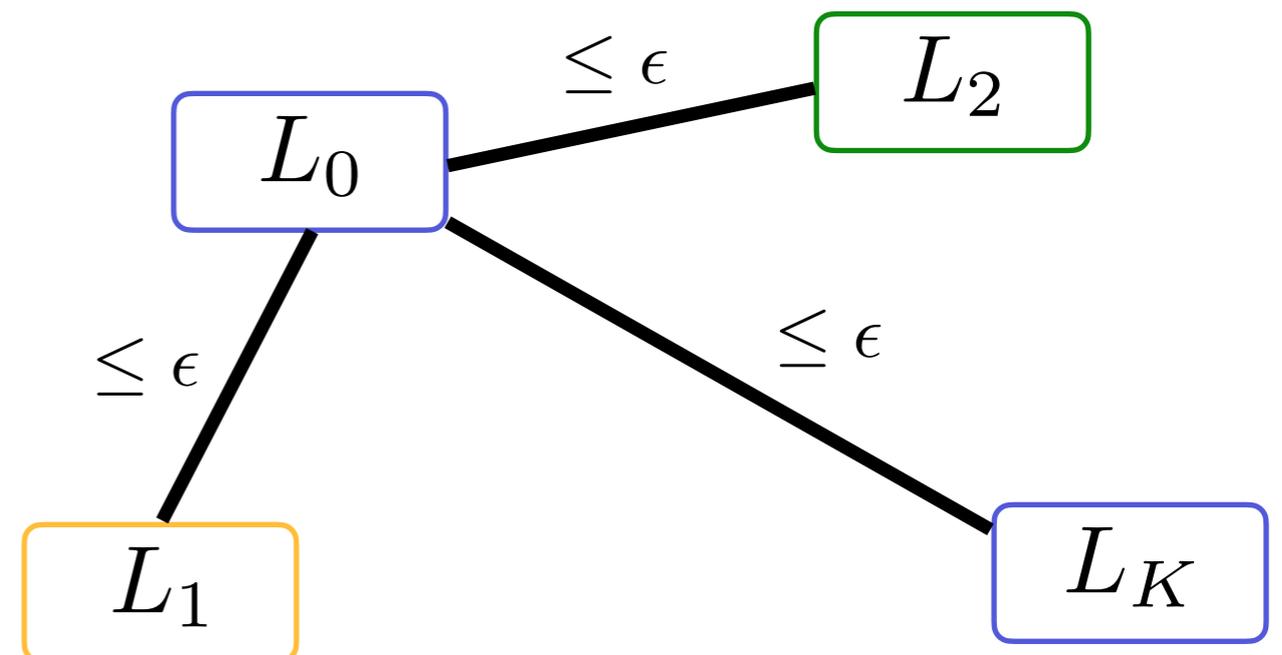


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- $\varepsilon(\hat{E}_i, \hat{E}_j)$ is measured w.r.t. the ground truth encoder-decoder
- ρ is the Lipschitz-constant of the ground truth encoder and decoder
- ϵ is the maximum error on each seen translation pair
- For a specified translation error ϵ , a corpora containing $O(1/\epsilon^2)$ parallel sentences suffices



We use an epsilon-net argument to prove this result

Summary

Without data generating assumption: An Impossibility Theorem, UMT has to incur a large error on at least one translation pair.

Theorem (informal): Consider a restricted setting of universal machine translation task with two source languages and one target language. If g is a universal language mapping, then for any decoder $h : \mathcal{Z} \rightarrow \Sigma_L^*$,

$$\text{Err}_{\mathcal{D}_0}^{L_0 \rightarrow L}(h \circ g) + \text{Err}_{\mathcal{D}_1}^{L_1 \rightarrow L}(h \circ g) \geq d_{\text{TV}}(\mathcal{D}_{L_0, L}(L), \mathcal{D}_{L_1, L}(L)).$$

With a natural data generating assumption:
Linear number of translation pairs suffices!

Theorem (informal): Let $\text{diam}(H)$ be the diameter of the translation graph H , then for any pair of language L_i, L_j , the translation error has the following upper bound:

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